International Journal of Theoretical Physics, Vol. 26, No. 1, 1987

CP-Nonconservation and Electric Dipole Moment of Fermions in the Nonsymmetric Kaluza–Klein Theory

M. W. Kalinowski[1]

Received September 3, 1986

A generalization of minimal coupling is proposed and the usual Dirac equation is generalized within the nonsymmetric Kaluza-Klein theory and the nonsymmetric Jordan-Thiry theory. The dipole electric moment of fermions of order 10^{-31} cm is obtained.

1. INTRODUCTION

In this paper we consider the Dirac equation in the nonsymmetric Kaluza-Klein theory (Kalinowski, 1983a,c, 1984a) and in the nonsymmetric Jordan-Thiry theory (Kalinowski, 1983b, 1984b). We introduce a generalization of the minimal coupling scheme between spinor fields and gravitational and electromagnetic fields. In this way, we get in the Lagrangian a term that describes an interaction between the dipole electric moment of fermions and the electromagnetic field from the nonsymmetric Kaluza-Klein theory. We also get a term that has a pseudomass character. Both terms break *PC* and *P*. The value of this dipole electric moment is the same as in symmetric theory (Thirring, 1972; Kalinowski, 1981a,b).

The paper is organized as follows. In Section 2 we introduce some elements of the nonsymmetric Kaluza-Klein theory and the nonsymmetric Jordan-Thiry theory. In Section 3 we describe the minimal coupling scheme in the nonsymmetric theory of gravitation. In Section 4 we introduce a generalization of minimal coupling for the Dirac field and discuss new effects appearing in our model, i.e., the dipole electric moment and pseudomass-like term.

[1]Department of Physics, University of Toronto, Toronto, Ontario, Canada M5S 1A7, and Institute of Experimental Physics, University of Warsaw, 00-681 Warsaw, Poland.

2. ELEMENTS OF THE NONSYMMETRIC KALUZA–KLEIN THEORY AND THE NONSYMMETRIC JORDAN–THIRY THEORY

Let us introduce the principal fiber bundle $\underline{P}$ over space-time E with the structure group $U(1)_{\text{el}}$ and with the projection π. We define on $\underline{P}$ a connection α (an electromagnetic connection). For a curvature Ω of α we have

$$\Omega = \pi^*(1/2F_{\mu\nu}\bar{\theta}^{\mu} \wedge \bar{\theta}^{\nu}) \tag{1}$$

where π^* is a horizontal lift and $F_{\mu\nu}$ is the electromagnetic field.

We introduce on $\underline{P}$ a frame

$$\theta^A = (\pi^*(\bar{\theta}^{\alpha}), \theta^5 = \lambda\alpha), \qquad \lambda = \text{const} \tag{2}$$

where $\bar{\theta}^{\alpha}$ is a frame on E. In the nonsymmetric Kaluza–Klein theory $\underline{P}$ is nonsymmetrically metrized in a natural way and has a linear connection ω^A_B which is compatible with this nonsymmetric metric. The metric tensor on $\underline{P}$ is as follows (Kalinowski, 1983a):

$$\gamma = \pi^*(g) - \theta^5 \otimes \theta^5 \tag{3}$$

where

$$g_{\alpha\beta} = g_{(\alpha\beta)} + g_{[\alpha\beta]} \tag{4}$$

is a nonsymmetric metric tensor on E [see Kalinowski (1983a–c, 1984a) for details]. The following notation is used in this paper. Capital italic indices $A, B, C = 1, 2, 3, 4, 5$; lower-case Greek indices $\alpha, \beta, \gamma = 1, 2, 3, 4$. The bar above ω^{α}_{β} indicates that $\bar{\omega}^{\alpha}_{\beta}$ is defined on E. Here $\bar{D}$ means the covariant exterior derivative with respect to $\bar{\omega}^{\alpha}_{\beta}$. Let us introduce a linear connection compatible with the nonsymmetric metric tensor γ, with

$$D\gamma_{A+B-} = D\gamma_{AB} - \gamma_{AD}Q^D_{BC}(\Gamma)\theta^C = 0 \tag{5}$$

where

$$\omega^A_B = \Gamma^A_{BC}\theta^C$$

D is the exterior covariant derivative with respect to the connection ω^A_B [see Kalinowski (1983a) for details] and $Q^D_{BC}(\Gamma)$ is the tensor of torsion for the connection ω^A_B.

The solution of (5) is as follows:

$$\omega^A_B = \left(\begin{array}{c|c} \pi^*(\bar{\omega}^{\alpha}_{\beta}) + \frac{1}{2}\lambda g^{\gamma\alpha}H_{\gamma\beta}\theta^5 & \frac{1}{2}\lambda H_{\beta\gamma}\theta^{\gamma} \\ \hline \frac{1}{2}\lambda g^{\alpha\beta}(H_{\gamma\beta} - 2F_{\beta\gamma})\theta^{\gamma} & 0 \end{array}\right) \tag{6}$$

where

$$H_{\beta\gamma} = -H_{\gamma\beta} \tag{7a}$$

$$g_{\delta\beta} g^{\gamma\delta} H_{\gamma\alpha} + g_{\alpha\delta} g^{\delta\gamma} H_{\beta\gamma} = 2 g_{\alpha\delta} g^{\delta\gamma} F_{\beta\gamma} \tag{7b}$$

$\bar{\omega}^{\alpha}_{\beta}$ is the connection on space-time E with the following properties:

$$\bar{D} g_{\alpha+\beta-} = \bar{D} g_{\alpha\beta} - g_{\alpha\delta} \bar{Q}^{\delta}_{\beta\gamma}(\bar{\Gamma}) \bar{\theta}^{\gamma} = 0, \qquad \bar{Q}^{\delta}_{\alpha\delta}(\bar{\Gamma}) = 0 \tag{8}$$

i.e., it is a second connection from Moffat's theory of gravitation (Moffat, 1979, 1981, 1982).

In the Kaluza-Klein theory we have $\lambda = 2G^{\frac{1}{2}}/c^2$, where G is a gravitational constant and c is the velocity of light in vacuum. This condition originates from the consistency between the equation in the Kaluza-Klein theory and Einstein's equation (see Kaluza, 1921, Lichnerowicz, 1955, Moffat, 1979, 1981). In the nonsymmetric Jordan-Thiry Theory we have in place of (3)

$$\gamma = \pi^*(g) - \rho^2, \ \theta^5 \otimes \theta^5 \tag{9}$$

where $\rho = \rho(x)$ is a scalar field on E. From (5) we get (see Kalinowski, 1981b)

$$\omega^{A}_{B} = \left[\begin{array}{c|c} \pi^*(\bar{\omega}^{\alpha}_{\beta}) + \frac{\lambda}{2}\rho^2 g^{\delta\alpha} H_{\delta\beta} \theta^5 & \frac{\gamma}{2} H_{\beta\gamma}\theta^{\gamma} + \frac{1}{\rho} g_{\beta\delta} \tilde{g}^{(\alpha\delta)} \rho_{,\alpha} \theta^5 \\ \hline \frac{\lambda}{2}\rho^2 g^{\alpha\beta}(H_{\gamma\beta} - 2F_{\gamma\beta})\theta^{\gamma} + \rho \tilde{g}^{(\gamma\alpha)} \rho_{,\gamma} \theta^5 & \frac{1}{\rho} g_{\delta\gamma} \tilde{g}^{(\alpha\delta)} \rho_{,\alpha} \theta^{\gamma} \end{array}\right] \tag{1.10}$$

where $\tilde{g}^{(\alpha\delta)}$ is the inverse tensor for $g_{(\alpha\beta)}$ i.e., $\tilde{g}^{(\alpha\delta)} g_{(\alpha\beta)} = \delta^{\delta}_{\beta}$ and $H_{\gamma\beta}$ satisfies conditions (7a) and (7b) (see Kalinowski, 1981b and 1983b for details). The connection $\bar{\omega}^{\alpha}_{\beta}$ satisfies the condition (8). In the Moffat theory of gravitation the connection $\bar{W}^{\alpha}_{\beta}$ is defined such that:

$$\bar{W}^{\alpha}_{\beta} = \bar{\omega}^{\alpha}_{\beta} - \tfrac{2}{3}\delta^{\alpha}_{\beta} \bar{W} \tag{11}$$

where

$$\bar{W} = \bar{W}_{\gamma} \bar{\theta}^{\gamma} = 1/2(W^{\sigma}_{\gamma\sigma} - W^{\sigma}_{\sigma\gamma})\theta^{\gamma} \tag{1.12}$$

(see Kalinowski, 1984b, 1984c, 1986). Let $\bar{D}_w$ mean a covariant exterior derivative with respect to $\bar{W}^{\alpha}_{\beta}$ on E.

On the manifold E, i.e., on space-time, we introduce a Levi-Civita symbol and Cartan dual base

$$\bar{\eta}_{\alpha\beta\gamma\delta}, \ \bar{\eta}_{1234} = (-\det g)^{1/2} \tag{13}$$

$$\bar{\eta}_{\alpha} = \frac{1}{2 \cdot 3} \bar{\theta}^{\delta} \wedge \bar{\theta}^{\gamma} \wedge \bar{\theta}^{\beta} \bar{\eta}_{\alpha\beta\gamma\delta} \tag{14}$$

$$\bar{\eta} = 1/4 \bar{\theta}^{\alpha} \wedge \bar{\eta}_{\alpha} \tag{15}$$

It is easy to see that we get the nonsymmetric Kaluza-Klein theory from the nonsymmetric Jordan-Thiry theory if $\rho \equiv 1$.

3. DIRAC LAGRANGIAN IN MOFFAT'S THEORY OF GRAVITATION

In Kalinowksi (1986) we found the minimal coupling scheme for the Dirac field in the Moffat theory of gravitation. We get the Lagrangian

$$L(W,\psi)=\tfrac{1}{2}i\hbar c(\bar{\psi}l\wedge\hat{D}\psi+\hat{D}\bar{\psi}\wedge l\psi)+mc\eta\bar{\psi}\psi \tag{16}$$

where $l=\gamma^{\mu}\bar{\eta}_{\mu}$ and

$$\begin{aligned}\hat{D}\psi&=D_{w}\psi-\tfrac{1}{3}in\varepsilon_{\mathrm{F}}(a/l_{\mathrm{Pl}})^{2}\bar{W}\psi\\ D\bar{\psi}&=D_{w}\bar{\psi}+\tfrac{1}{3}in\varepsilon_{\mathrm{F}}(a/l_{\mathrm{Pl}})^{2}\bar{W}\bar{\psi}\end{aligned} \tag{17}$$

where a is a coupling constant for fermion current in the Moffat theory (Moffat, 1979, 1981, 1982), l_{Pl} is the Planck length, $l_{\mathrm{Pl}}=(G/\hbar c)^{1/2}\approx 10^{-33}$ cm, n is a nonzero integer, and $\varepsilon_{\mathrm{F}}^{2}=1$. In Kalinowski (1986) we proved that the Lagrangian (17) is equal to

$$\begin{aligned}L(W,\psi)=\tfrac{1}{2}i\hbar c[&\bar{\psi}l\wedge(\bar{D}\psi-\tfrac{1}{3}in\varepsilon_{\mathrm{F}}\bar{W}\psi)\\ &+(\bar{D}\bar{\psi}+\tfrac{1}{3}in\varepsilon_{\mathrm{F}}\bar{W}\bar{\psi})\wedge l\psi]+mc\eta\bar{\psi}\psi\end{aligned} \tag{18}$$

where

$$\bar{D}\psi=d\psi+\omega^{\alpha}_{\beta}\sigma^{\beta}_{\alpha}\psi,\qquad \bar{D}\bar{\psi}=d\bar{\psi}-\bar{\psi}\sigma^{\beta}_{\alpha}\omega^{\alpha}_{\beta} \tag{19}$$

and σ^{β}_{α} satisfies the following properties:

$$\sigma^{\beta}_{\beta}=0 \tag{20}$$

$$2[\sigma^{\mu}_{\nu},\sigma^{\kappa}_{\lambda}]=\delta^{\kappa}_{\nu}\sigma^{\mu}_{\lambda}-\eta_{\nu\lambda}\sigma^{\mu\kappa}+\delta^{\mu}_{\lambda}\sigma^{\kappa}_{\nu}-\eta^{\mu\kappa}\sigma_{\nu\lambda} \tag{21}$$

$$[\sigma^{\mu}_{\nu},\gamma^{\rho}]=1/2(\delta^{\rho}_{\nu}\gamma^{\mu}-\eta^{\rho\mu}\gamma_{\nu}) \tag{22}$$

γ^{μ} are ordinary Dirac matrices satisfying the conventional relationships,

$$\{\gamma^{\mu},\gamma^{\nu}\}=2\eta^{\mu\nu} \tag{23}$$

$$\begin{aligned}\eta_{\nu\lambda}\eta^{\lambda\kappa}&=\delta^{\kappa}_{\nu},\qquad \gamma^{\mu}=\eta^{\alpha\beta}\gamma_{\beta}\\ \sigma^{\mu\kappa}&=\eta^{\kappa\nu}\sigma^{\mu}_{\nu},\qquad \sigma_{\nu\lambda}=\eta_{\nu\mu}\sigma^{\mu}_{.\lambda}\end{aligned} \tag{24}$$

The contragredient spinor $\bar{\psi}$ is defined by

$$\bar{\psi}=\psi^{+}\beta,\qquad \beta^{+}=\beta \tag{25}$$

where

$$\gamma^{\mu+}=\beta\gamma^{\mu}\beta^{-1} \tag{26}$$

and

$$\sigma^{\mu+}_{.\lambda}=-\beta\sigma^{\mu}_{\lambda}\beta^{-1} \tag{27}$$

The superscript plus sign denotes Hermitian conjugation. The spinor ψ was defined in Kalinowski (1986) as a 0-form of Σ-type:

$$\Sigma:\quad GL(4,R)\to GL(4,C)\qquad [\text{or } GL_{\downarrow}(4,R)] \tag{28}$$

and

$$\sigma^{\mu}{}_{\nu\tilde{B}}^{\tilde{A}} = (\partial\Sigma_{\tilde{B}}^{\tilde{A}}/\partial A_{\mu}^{\nu})|_{A_{\mu}^{\nu}=\delta_{\mu}^{\nu}} \tag{29}$$

[see Kalinowski (1986) for more details]. It is easy to see that

$$\sigma_{\lambda}^{\mu} = \tfrac{1}{8}(\gamma^{\mu}, \gamma_{\lambda}) \tag{30}$$

satisfies all properties (20–22). In Kalinowski (1986) we proved that the Lagrangian (16) or (17) has $U(1)_F$-gauge invariance, which is connected to a compactification of the dilatation subgroup R_+ of $GL_\downarrow(4, R) = R_+ \otimes SL(4, R)$, where

$$\begin{aligned} GL_\downarrow(4, R) &= \{A \in GL(4, R), \det A > 0\} \\ SL(4, R) &= \{A \in GL(4, R), \det A = 1\} \end{aligned} \tag{31}$$

$R_+ = \{e^\rho, \rho \in R\}$, where $\rho = \ln(\det A)$, and R_+ acts in the following way on ψ, $\bar{\psi}$, and $\bar{W}$:

$$\bar{W} \to \bar{W}' = \bar{W} + d\phi, \qquad \phi = -\tfrac{3}{8}\ln(\det A) \tag{32a}$$

$$\psi \to \psi' = \exp\left[i\frac{n\varepsilon_F}{8\hbar c}\left(\frac{a}{l_{\rm Pl}}\right)^2 \ln(\det A)\right]\psi \tag{32b}$$

$$\bar{\psi} \to \bar{\psi}' = \exp\left[-i\frac{n\varepsilon_F}{8\hbar c}\left(\frac{a}{l_{\rm Pl}}\right)^2 \ln(\det A)\right]\bar{\psi} \tag{32c}$$

4. DIRAC EQUATION IN THE NONSYMMETRIC KALUZA–KLEIN THEORY AND IN THE NONSYMMETRIC JORDAN–THIRY THEORY

In this section we deal with a generalization of the Dirac equation on a manifold $\underline{P}$ (a nonsymmetrically metrized electromagnetic bundle). The results we obtain are similar to Thirring's (1972) results in the symmetric case and similar to the results from Kalinowski (1981b). We introduce several kinds of derivatives, and using them, we get a generalization of the Dirac equation. We define the spinor fields ψ, $\bar{\psi}$, $\psi: E \to C^4$ on E and spinor fields Ψ, $\bar{\Psi}$ on $\underline{P}$, $\Psi: P \to C^4$. For Ψ and $\bar{\Psi}$ we have

$$\begin{aligned} \psi(pg) &= \sigma(g^{-1})\psi(p), \qquad \sigma \in L(C^4) \\ \bar{\psi}(pg) &= \bar{\psi}(p)\sigma(g) \end{aligned} \tag{33}$$

where $p = (x, g_1) \in \underline{P}$; $g_1, g \in U(1)_{\rm el}$. Spinor fields ψ and $\bar{\psi}$ on E are defined modulo a phase factor and should be written ψ^f, $\bar{\psi}^f$ rather than ψ, $\bar{\psi}$ (f is a section of a bundle $\underline{P}$, $f: E \to \underline{P}$). Thus obviously we have

$$\begin{aligned} \Psi(f(x)) &= \pi^*(\psi^f(x)), \qquad \psi^f = f^*\Psi \\ \bar{\Psi}(f(x)) &= \pi^*(\bar{\psi}^f(x)), \qquad \bar{\psi}^f = f^*\bar{\Psi} \end{aligned} \tag{34}$$

Let us define an electromagnetic gauge derivative d_1 of the field Ψ:

$$d_1\bar{\psi} = \text{hor } d\Psi \tag{35}$$

(horizontality is understood in the sense of the electromagnetic connection α on $\underline{P}$).

Let us define a matrix γ^5

$$\gamma^5 = \gamma^1\gamma^2\gamma^3\gamma^4 \tag{36}$$

$$(\gamma^5)^2 = -1 \tag{37}$$

It is easy to see that

$$\{\gamma^A, \gamma^B\} = 2\bar{g}^{AB} \tag{38}$$

$A, B = 1, 2, 3, 4, 5$, where $\gamma^A = (\gamma^\alpha, \gamma^5)$ and $\bar{g}^{AB} = \text{diag}(-1, -1, -1, +1, -1)$. We also have

$$\gamma^5 = \beta\gamma^5\beta^{-1}, \qquad \bar{\Psi} = \Psi^+\beta \tag{39}$$

so

$$\gamma^{A+} = \beta\gamma^A\beta^{-1} \tag{40}$$

We perform an infinitesimal change of the frame θ^A

$$\theta'^A = \theta^A + \delta\theta^A = \theta^A - \varepsilon^A_B\theta^B \tag{41}$$

If the spinor field Ψ corresponds to θ^A and Ψ' to θ'^A, then we have

$$\begin{aligned} \Psi' &= \Psi + \delta\bar{\psi} = \Psi - \varepsilon^A_B\hat{\sigma}^B_A\bar{\Psi} \\ \bar{\Psi}' &= \bar{\Psi} + \delta\bar{\Psi} = \bar{\Psi} + \bar{\Psi}\hat{\sigma}^B_A\varepsilon^A_B \end{aligned} \tag{42}$$

where

$$\hat{\sigma}^{B}_{A}{}^{\tilde{A}}_{\tilde{B}} = \left.\frac{\partial\hat{\Sigma}^{\tilde{A}}_{\tilde{B}}}{\partial A^A_B}\right|_{A^A_B = \delta^A_B} \tag{43}$$

$$\hat{\Sigma}\colon \quad GL(5, R) \to GL(4, C) \tag{44}$$

is a homomorphsim of Lie groups and $\hat{\sigma}^B_A$ is the differential of $\hat{\Sigma}$ at the unit element. $\hat{\sigma}^B_A$ satisfies the following properties similar to σ^β_α [see (20)-(22)]:

$$\hat{\sigma}^A_A = 0 \tag{45}$$

$$2(\hat{\sigma}^B_A, \hat{\sigma}^D_C) = \delta^D_A\hat{\sigma}^B_C - \bar{g}_{AC}\hat{\sigma}^{BD} + \delta^B_C\hat{\sigma}^D_A - \bar{g}^{BD}\hat{\sigma}_{AC} \tag{46}$$

$$(\hat{\sigma}^B_A, \gamma^C) = 1/2(\delta^C_A\gamma^B - \bar{g}^{CB}\gamma_A) \tag{47}$$

where

$$\bar{g}_{AB}\bar{g}^{BC} = \delta^C_A, \qquad \gamma^A = \bar{g}^{AB}\gamma_B$$

$$\hat{\sigma}^{AB} = \bar{g}^{AC}\hat{\sigma}^B_C, \qquad \hat{\sigma}_{AB} = \bar{g}_{CB}\hat{\sigma}^C_A$$

We can use the following representation of $\hat{\sigma}^B_A$:

$$\hat{\sigma}^B_A = \tfrac{1}{8}[\gamma_A, \gamma^B] \tag{48}$$

Notice that the dimension of the spinor space for a $2n$-dimensional space is 2^n and it is the same for a $(2n+1)$-dimensional space. We take a spinor field for a five-dimensional space $\underline{P}$ and assume that the dependence on the fifth dimension is trivial, i.e., equation (33) holds. Taking a section, we get spinor fields on E (of the same dimension as the spinor space). In the case of an arbitrary gauge group G the situation is more complicated—after projecting on E we obtain several different spinor fields. Let us consider a covariant derivative of spinor fields Ψ and $\bar{\Psi}$ on $\underline{P}$,

$$D\Psi = d\Psi + \omega^A_B \hat{\sigma}^B_A \Psi, \qquad D\bar{\Psi} = d\bar{\Psi} - \bar{\Psi}\hat{\sigma}^B_A \omega^A_B \tag{49}$$

with respect to the linear connection ω^A_B from the nonsymmetric Kaluza–Klein theory [equation (6)] or from the nonsymmetric Jordan–Thiry theory [see equation (10)].

Now we introduce derivatives $\mathbf{D}$, i.e., "gauge" derivatives of a new kind (similar to those in Kalinowski (1981a,b, 1982, 1983d, 1984c). These derivatives may be treated as a generalization of minimal coupling between the spinor and electromagnetic fields on $\underline{P}$:

$$\mathbf{D}\Psi = \text{hor } D\Psi, \qquad \mathbf{D}\bar{\Psi} = \text{hor } D\bar{\Psi} \tag{50}$$

(horizontality is understood in the sense of electromagnetic connection α on $\underline{P}$). Using (6) and (10), one gets

$$\begin{aligned} \mathbf{D}\Psi &= \bar{\mathbf{D}}\Psi - \tfrac{1}{8}\lambda[g^{\beta\alpha}(H_{\gamma\alpha} - 2F_{\gamma\alpha})\gamma_\beta + H_{\beta\gamma}\gamma^\beta]\gamma^5\Psi\theta^\gamma \\ \mathbf{D}\bar{\Psi} &= \bar{\mathbf{D}}\bar{\Psi} + \tfrac{1}{8}\lambda\bar{\Psi}[g^{\beta\alpha}(H_{\gamma\alpha} - 2F_{\gamma\alpha})\gamma_\beta + H_{\beta\gamma}\gamma^\beta]\gamma^5\theta^\gamma \end{aligned} \tag{51}$$

in the nonsymmetric Kaluza–Klein theory or

$$\begin{aligned} \mathbf{D}\Psi &= \bar{\mathbf{D}}\Psi - \tfrac{1}{8}\lambda\rho^2[g^{\beta\alpha}(H_{\gamma\alpha} - 2F_{\gamma\alpha})\gamma_\beta + H_{\beta\gamma}\gamma^\beta]\gamma^5\Psi\theta^\gamma \\ \mathbf{D}\bar{\Psi} &= \bar{\mathbf{D}}\bar{\Psi} + \tfrac{1}{8}\lambda\rho^2\bar{\Psi}[g^{\beta\alpha}(H_{\gamma\alpha} - 2F_{\gamma\alpha})\gamma_\beta + H_{\beta\gamma}]\gamma^5\theta^\gamma \end{aligned} \tag{52}$$

in the nonsymmetric Jordan–Thiry theory, where

$$\bar{\mathbf{D}}\Psi = \text{hor } \bar{D}\Psi, \qquad \bar{\mathbf{D}}\bar{\Psi} = \text{hor } \bar{D}\bar{\Psi} \tag{53}$$

The derivative $\bar{\mathbf{D}}$ is at the same time a covariant derivative with respect to both $\bar{\omega}^\alpha_\beta$ and the "gauge". It introduces an interaction between the electromagnetic and graviational fields with Dirac spinors in the classical way in general relativity or in the Einstein–Cartan theory (Trautman, 1973). Now let us turn to the Lagrangian (18) and lift it on the manifold $\underline{P}$. In order to do this we have to pass from $\bar{D}$ to $\mathbf{D}$ and from ψ, $\bar{\psi}$ to Ψ, $\bar{\Psi}$. In this case the Dirac Lagrangian takes the form

$$L = \tfrac{1}{2}i\hbar c[\bar{\Psi} l \wedge (\mathbf{D}\Psi - \tfrac{1}{3}in\varepsilon_F \bar{W}\Psi) + (\mathbf{D}\bar{\Psi} + \tfrac{1}{3}in\varepsilon_F \bar{W}\bar{\Psi}) \wedge l\Psi] + mc\bar{\eta}\bar{\Psi}\Psi \tag{54}$$

After some algebra one gets

$$
\begin{aligned}
L = L(\Psi, \bar{W}, \alpha) &- il_{\mathrm{Pl}}\tilde{\alpha}^{-1/2} q H_{\beta\alpha} \bar{\Psi}\gamma^5\sigma^{\beta\alpha}\Psi\bar{\eta} \\
&+ il_{\mathrm{Pl}}\tilde{\alpha}^{-1/2} q g^{\beta\alpha}(H_{\mu\alpha} - 2F_{\mu\alpha})\bar{\Psi}\gamma^5\sigma^{\mu}_{.\beta}\Psi\bar{\eta} \\
&- \tfrac{1}{8} il_{\mathrm{Pl}}\tilde{\alpha}^{-1/2} q (g^{[\beta\alpha]}F_{\beta\alpha})\bar{\Psi}\gamma^5\Psi\bar{\eta}
\end{aligned} \tag{55}
$$

where l_{Pl} is the Planck length, q is the elementary charge, $\tilde{\alpha}$ is the fine structure constant, and

$$
\begin{aligned}
L(\Psi, W, \alpha) = \tfrac{1}{2} i\hbar c[\bar{\Psi} l \wedge (\bar{\mathbf{D}}\Psi - \tfrac{1}{3} in\varepsilon_{\mathrm{F}} \bar{W}\Psi) \\
+ (\bar{\mathbf{D}}\bar{\Psi} + \tfrac{1}{3} in\varepsilon_{\mathrm{F}} \bar{W}\bar{\Psi}) \wedge l\Psi] + mc\bar{\eta}\bar{\Psi}\Psi
\end{aligned} \tag{56}
$$

$L(\Psi, W, \alpha)$ describes the interaction between the spinor field geometry in the nonsymmetric theory of gravitation and the electromagnetic field [as in Kalinowski (1986)]. In the case of the nonsymmetric Jordan–Thiry theory we get

$$
\begin{aligned}
L = L(\Psi, W, \alpha) &- il_{\mathrm{Pl}}\alpha^{-1/2} g\rho^2 H_{\beta\alpha} \bar{\Psi}\gamma^5\sigma^{\beta\alpha}\Psi\bar{\eta} \\
&+ il_{\mathrm{Pl}}\alpha^{-1/2} g\rho^2 g^{\beta\alpha}(H_{\mu\alpha} - 2F_{\mu\alpha})\bar{\Psi}\gamma^5\sigma^{\mu}_{.\beta}\Psi\bar{\eta} \\
&- \tfrac{1}{8} il_{\mathrm{Pl}}\alpha^{-1/2} g\rho^2 (g^{[\beta\alpha]}F_{\beta\alpha})\bar{\Psi}\gamma^5\Psi\bar{\eta}
\end{aligned} \tag{57}
$$

so we see that we get additional terms. They are

$$
il_{\mathrm{Pl}}\alpha^{-1/2} q[g^{\beta\alpha}(H_{\mu\alpha} - 2F_{\mu\alpha})\bar{\Psi}\gamma^5\sigma^{\mu}_{\beta}\Psi - H_{\beta\alpha}\bar{\Psi}\gamma^5\sigma^{\beta\alpha}\Psi]\bar{\eta} \tag{58}
$$

and

$$
- il_{\mathrm{Pl}}\alpha^{-1/2} q (g^{[\beta\alpha]}F_{\beta\alpha})\bar{\Psi}\gamma^5\Psi\bar{\eta} \tag{59}
$$

The first term (58) describes the interaction between the dipole electric moment of fermions with the electromagnetic field, which is represented by tensors $H_{\alpha\beta}$ and $F_{\alpha\beta}$ [see equations (7a) and (7b); for more details see Kalinowski (1973a)]. This is similar to the symmetric Kaluza–Klein theory (Thirring, 1972; Kalinowski, 1981a,b). However, now, due to the skew-symmetric part of the metric on the space-time E, this term is more complicated. The second term exists only due to the skew-symmetric part of the metric and is zero if $g_{[\alpha\beta]} = 0$. It describes a parity-breaking interaction (pseudomass-like term). In the case of the Jordan–Thiry theory we get the same terms in (57) as in (55). The only difference is that here we also get an interaction with the scalar field ρ. This field is connected with the gravitational constant in the Jordan–Thiry theory. This results in the effective value of the dipole electric moment of fermions changing according to this field. If the skew-symmetric part of the metric is zero, we get the results of Kalinowski (1981b). In the same way as in Kalinowski (1981b) we can introduce operators of discrete transformations on $\underline{P}$: Π (space reflection),

T (time reversal), C (charge conjugation), and the combined transformation ΠC, $\theta = \Pi CT$. In the same way as in Kalinowski (1981b) we get nonconservation of PC and T due to the dipole electric moment of fermions. After projection on E we get ordinary operators of these transformations, well known in the literature. The value of our dipole electric moment is of course the same,

$$l_{\mathrm{Pl}}\alpha^{-1/2}q \approx 10^{-31}q \text{ cm} \tag{60}$$

The second term (59) breaks parity and this breaking is of the same order as (60). We consider all of these effects as "interference effects" between gravitational and electromagnetic fields from the nonsymmetric Kaluza-Klein theory. In the nonsymmetric Kaluza-Klein theory and Jordan-Thiry theory we also have different "interference effects" between the gravitational and electromagnetic fields (Kalinowski, 1983a-c; 1984a,b).

ACKNOWLEDGMENTS

I would like to thank Prof. J. W. Moffat and Drs. G. Kunstatter and R. B. Mann for their kind hospitality during my stay at the Physics Department of the University of Toronto and for many stimulating discussions. I would like to thank Ms. A. Sousa for typing my manuscripts.

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